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Dynamics and kinematics of complex mechanical systems harnessing multibody dynamic program

Mykhailo Kaidash[1], Serhii Selevych[2]

[1]Department of Welding and Construction Technologies, Chernihiv Polytechnic National University, Chernihiv, Ukraine
[2]Department of Car and Tractor Industry, Institute of Education and Science in Mechanical Engineering and Transport, National Technical University (Kharkiv Polytechnic Institute), Kharkiv, Ukraine

ABSTRACT

Understanding the behavior and performance of engineering applications like machines, transport machines, manipulators, and mechanisms like gears relies heavily on the study of the dynamics and kinematics of complex mechanical systems. This article provides a comprehensive overview of recent findings and advancements in this field. The purpose of this work is to provide an in-depth introduction to the theoretical and practical considerations involved in assessing the dynamic and kinematic properties of such complex systems. Understanding forces, torques, displacements, and velocities is highlighted as crucial to the design and study of complex mechanical systems, and the underlying mathematical models and concepts that control their motion are investigated. This paper also evaluates and critiques the most current developments in modeling and simulation approaches such as finite element analysis (FEA), computational dynamics, and optimization strategies. The multidisciplinary aspect of the topic and its potential to progress numerous engineering, robotics, and industrial applications constitute the topic's scientific uniqueness. The results include various advanced modeling and simulation techniques like FEA, computational dynamics, and multibody dynamics simulation. In conclusion, this article compiles a lot of information on the dynamics and kinematics of sophisticated mechanical systems, such as machines, transport machines, manipulators, and mechanisms.

Corresponding Author:

Mykhailo Kaidash
Department of Welding and Construction Technologies, Chernihiv Polytechnic National University
Chernihiv, Ukraine
Email: 0203kmdcn@gmail.com

1. INTRODUCTION

One of the most important subfields of mechanical engineering is the study of the dynamics and kinematics of complex mechanical systems. Understanding the dynamic and kinematic properties of these systems is crucial to their design, performance, and optimization, and this applies to everything from the complex interaction of gears in machines to the precise movements of manipulators and the flawless working of transport machines. Through an exploration of the approaches, problems, and achievements that have molded the evolution of this diverse topic, this review article aims to offer a complete analysis of the current state-of-the-art research and innovations in the area. Business players can benefit from the study's findings when it comes to using venture capital and innovation in products. When resources (both financial and human) are few, innovation cloning becomes necessary [1]. To lay the groundwork for making good use of different kinds of data for company growth, contemporary innovation management might employ business

intelligence technologies [2]. There has been a meteoric rise in the complexity and variety of mechanical systems available for use in today's factories, vehicles, and robots. The need for a comprehensive knowledge of the dynamics and kinematics driving these systems has increased as the pursuit of greater efficiency, safety, and dependability has gained momentum. Engineers may improve system performance and come up with novel designs by dissecting the intricate relationships between motion, forces, and torques. Machines, transport machines, manipulators, and mechanisms are just some of the many mechanical systems that might benefit from kinematic and dynamic analysis. The first part of this review article introduces the reader to the fundamental ideas of dynamics and kinematics, setting the stage for a thorough examination of their applications. Displacement, velocity, acceleration, and the driving forces are all highlighted to help readers better understand the mathematical models and concepts that control the motion of these complex systems.

The aim of this study is to broaden the understanding of the dynamics and kinematics of complex mechanical systems, encompassing machines, intricate mechanisms. By exploring the fundamentals of the advanced techniques, the research really considers to be more qualified than others. By addressing the gaps and key challenges the research aims really got the efficiency and reliability of complex mechanical systems in different genres of industries. Some research questions to address the topic of this paper are: i) how do the kinematics and dynamics influence the behavior and performance of machines and manipulators?; ii) what are the key factors associated with the nonlinearity of complex mechanical systems?; iii) what are the advanced modeling and simulation techniques?; iv) what methods are employed to guide the uncertainties and variations in both modeling and real-world applications?; and v) what are the results variations considering dynamic analysis of generic planar multibody systems?.

As a space manipulator should be as light as possible to lower its launching cost, book [3], [4], research on the control of a flexible arm manipulator began as part of the study of space robotics. Uchiyama *et al.* [5] suggested astronautical dual arm manipulator (ADAM) to understand the vibration link in an effective way. Alberts *et al.* [6] used modal potential energy analysis to understand the dominant contributors to the end-point. Krishnamurthy and Chao [7] considered active vibration with two different control strategies. On the contrary, Dubowsky [8] suggested a very promising field systems for long reach manipulator system. Cyril *et al.* [9] used the Lagrangian formulation to derice the dynamical equations of the system considerably. Mavroidis *et al.* [10] was another author to develop the long reach manipulator systems carried by a deployable structure. Nagaraj *et al.* [11] and Gouliaev and Zavrazhina [12] also studied flexible manipulators used for space applications. Dynamic control of lightweight robots is difficult, and Shi *et al.* [13] emphasised this as a problem for both space and terrestrial applications.

However, model building and, by extension, assumptions and approximations underpins all research. Faster data processing capabilities encouraged more precise procedures and techniques, which in turn led to enhancements to previously formulated models. In the last several decades, advancements in data processing speed have encouraged further study of complicated multibody systems. Forces in motion, including kinematics and dynamics, in multibody systems: analysis, synthesis, and optimisation [14], [15].

A kinematic chain's elastic link components move in several directions at once, and the gyroscopic interaction between their rotational and linear components makes it difficult to isolate and separate these movements for study. Examples of how they may be used to calculate an elastic one-link manipulator with a concentrated mass at the tip are provided. References take into account elastic models of multi-link manipulators for cosmic basis [12]. Regarding the vibration and dynamic stability of axially moving materials, Wickert and Mote [16] have published an outstanding study examining recent findings, prior work, and some ideas for future research. However, recently, a few flexible manipulators with more than three degrees-of-freedom have been developed. The issues with controlling the elastic robot's dynamics and kinematics are established. The method created for solving specific problems is based on the co-use of the initial parameter method [17], [18], the discrete orthogonalization algorithm, the implicit Houbolt scheme for nite differences, and the one- and multi-step-by-step methods of numerical integration [19].

2. METHODS OF DYNAMIC AND KINEMATICS

Machines, transport machines, manipulators, flat and spatial mechanisms, gears, and gear trains are only some of the sophisticated mechanical systems that may benefit from a knowledge of dynamics and kinematics. This part of the review article goes into the fundamental ideas of dynamics and kinematics, giving a broad overview of the theoretical concepts and mathematical methods used to analyze the motion and behavior of such complex systems. The study of mechanical systems' responses to forces, torques, and motion is known as dynamics, a subfield of mechanics. The purpose of dynamics analysis is to get a better comprehension of the stability, vibration, and behavior of complex mechanical systems by predicting how they will react to external forces and torques.

2.1. Newton's law of motion

The foundation of dynamics is found in Newton's three laws of motion. The first law asserts that, absent an intervening force, an object at rest will stay at rest and an object in motion will continue to move at a constant velocity. The second rule states that any force applied to an object will result in an equal and opposite acceleration of that object, usually expressed as F=ma. For every action, there is an equal and opposite response, as stated by Newton's third law of motion.

2.2. Angular and linear kinematics

The study of how things move in relation to their angular velocities and angular velocities is known as "angular kinematics." The study of motion in terms of linear displacements, velocities, and accelerations is what linear kinematics is all about. Understanding the behavior of complicated mechanical systems with rotating components requires taking into account the interaction of angular and linear motion.

2.3. Inertial properties

Center of mass, moment of inertia, and mass are all inertial qualities that are crucial to the study of dynamics. The moment of inertia may be used to quantify an object's resistance to rotational acceleration, whereas mass defines the resistance to linear acceleration when exposed to an external force. To grasp how an item moves in a straight line, it's crucial to locate its center of mass.

When studying motion, kinematics ignores the forces that produce it. Kinematic analysis is crucial for complex mechanical systems because it allows for the characterization of motion, the comprehension of range of motion, and the determination of relative locations and orientations of distinct elements. Determining the position, velocity, and acceleration of parts of a system is called a kinematic analysis. The term "velocity" indicates the rate of change of position relative to time, whereas "displacement" represents the change in position itself. The rate of change in velocity is quantified by the term "acceleration." Forward kinematics is used to determine the end-effector's location and orientation from known joint angles in robotic manipulators and spatial mechanisms. However, inverse kinematics entails working backwards from the intended end-effector location and orientation to determine the necessary joint angles. In manipulator kinematics, the Jacobian matrix is crucial in linking the joint velocities to the linear and angular velocities of the end-effector. The Jacobian is necessary for studying the velocity and singularity features of the manipulator.

3.   METHODS
3.1. Gear dynamics and kinematics

In order to transfer motion and power between two spinning shafts, gears are essential mechanical components. For effective power transfer, less wear, and enhanced overall performance and dependability in complex mechanical systems, knowledge of the dynamics and kinematics of gears is crucial. Gear kinematics is the study of the motion and interactions between gear pairs when they mesh. It delves into how the meshing and separating of gear teeth during rotation facilitates the transmission of force. Gear motion is explained theoretically by discussing the pitch circle, pitch point, and pitch line. The effect tiveness and dependability of a gear system are both impacted by the forces at play during gear meshing. Torque transmission is discussed in relation to the tangential force (F_t) and radial force (F_r) exerted between meshing gears. Gear ratio (GR) is discussed together with the effect gear shape has on meshing forces [20]. Gears must have enough backlash, or space, between their mating teeth to engage smoothly and prevent jamming [21]. The noise, wear, and efficiency losses caused by gear vibrations are all too real. In order to guarantee dependable gear performance, this section delves into vibration analysis methods such as modal analysis and dynamic response simulation. Fr, the radial force between gear teeth, is determined by the equation:

$$F_r = \frac{\tau * tan(\phi)}{d}$$

Where τ is the transmitted torque, d is the pitch diameter of the gear, and ϕ is the pressure angle.

To take into consideration the impact of dynamic loads on gear strength, the dynamic factor (K_v) is used. This number may be determined by using the formula:

$$K_v = \frac{10}{10+(\sqrt{T}+1.25)}$$

where T is the number of meshing teeth.

3.2. Dynamic equations of motion

Newton-Euler equations and Lagrange's equations may both be used to explain the dynamics of mechanical systems. For a 2-DOF machine with masses (m_1, m_2) and lengths (l_1, l_2), the equations of motion are as (1) and (2):

$$m_1 * (l_1^2) * \theta_1'' + m_2 * l_1 * l_2 * \theta_2'' * cos(\theta_2 - \theta_1) + m_2 * (l_2^2) * \theta_2'' = \tau_1 \qquad (1)$$

$$m_2 * l_1 * l_2 * \theta_1'' * cos(\theta_2 - \theta_1) + m_2 * (l_2^2) * \theta_2'' = \tau_2 \qquad (2)$$

where θ_1'' and θ_2'' represent the angular accelerations, and τ_1 and τ_2 are the applied torques.

Figure 1 illustrates the four bar linkage for motion generation within the area of dynamic motion consideration. The equations are completely supports the motion generation with the angular effects.
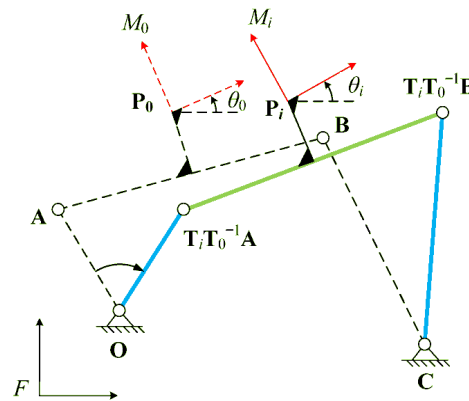

Figure 1. Four bar linkage for motion generation

GR calculation: when two gears of different tooth counts (N_1 and N_2) mesh, the GR between them may be found by using the formula:

$$GR = \frac{N_2}{N_1}$$

The link between gear rotational speeds and torques is expressed quantitatively here.

Investigating the meshing forces and torque gearbox is a common part of the dynamic study of gears. When two gears mesh, they exert a tangential force (Ft) and a radial force (Fr), respectively.

$$F_r = \frac{2 * \tau}{d * P} \qquad (3)$$

$$F_r = \frac{\tau * tan(\phi)}{d} \qquad (4)$$

Where τ is the transmitted torque, d is the gear pitch diameter, P is the circular pitch, and ϕ is the pressure angle.

3.3. Kinematic equations for a robot manipulator

Homogeneous transformation matrices may be used to depict the forward kinematics of a robot manipulator. Figure 2 schematic showing the movement of an asymmetrical crank and slider where kinematics distribution fulfilled its cycle with the mentioned equations. The end-effector position (x, y) may be represented as for a 3-link planar robot with joint angles 1, 2, and 3:

$$x = l_1 * cos(\theta_1) + l_2 * cos(\theta_1 + \theta_2) + l_3 * cos(\theta_1 + \theta_2 + \theta_3) \qquad (5)$$

$$y = l_1 * sin(\theta_1) + l_2 * sin(\theta_1 + \theta_2) + l_3 * sin(\theta_1 + \theta_2 + \theta_3) \qquad (6)$$

By differentiating the obtained dependence, the velocity analogues diagram for point B may be built. In the real world, people employ graphical methods of differentiation and integration. Motion diagram also can be suggested by graphical differentiation facts by chord method which is illustrated in the Figure 3.
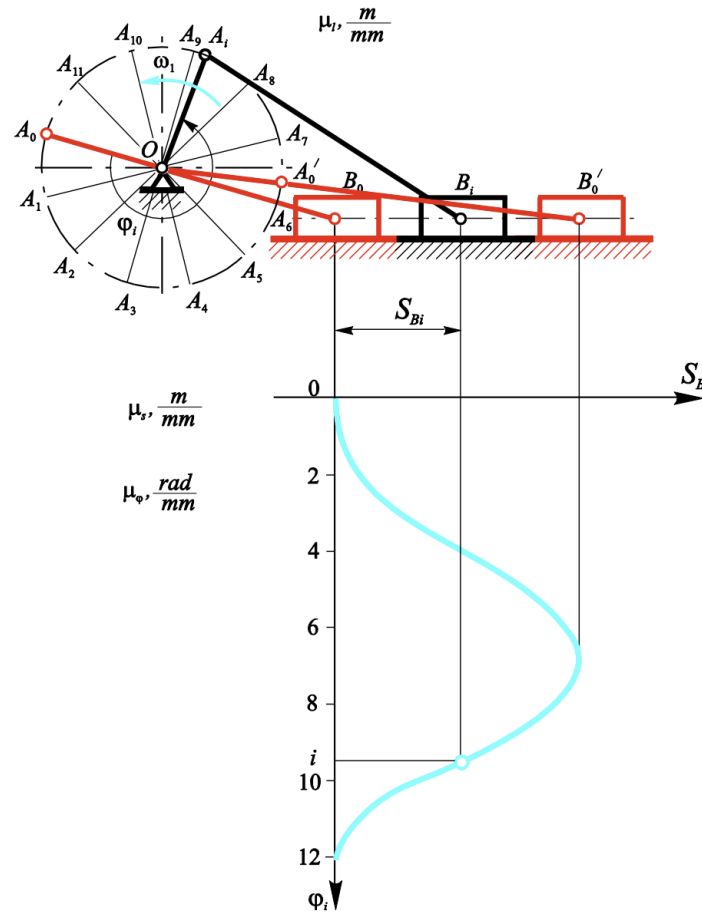
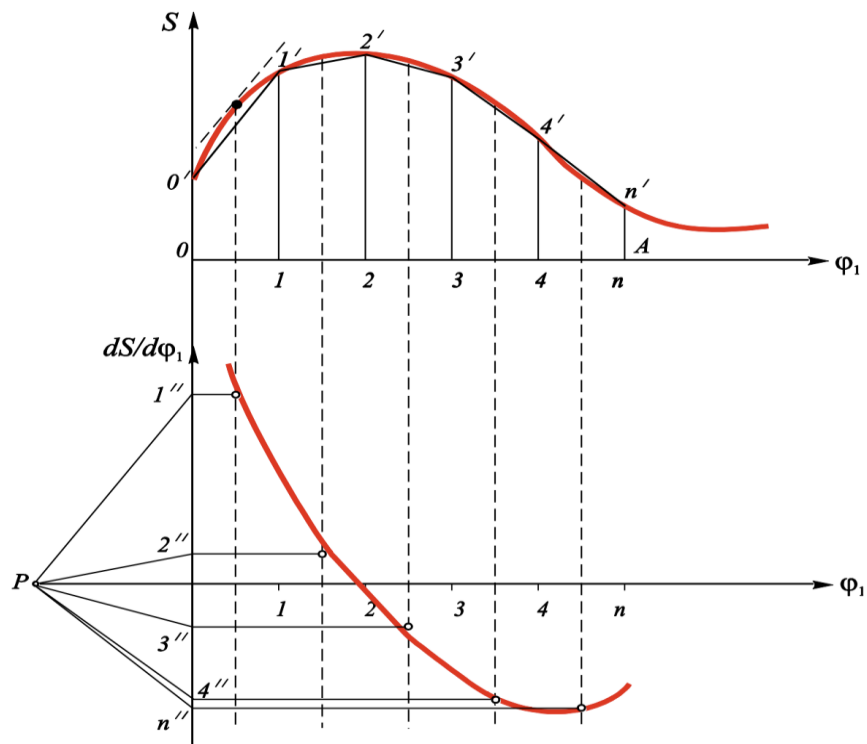Figure 2. Schematic showing the movement of an asymmetrical crank and slider [22]

Figure 3. Graphical differentiation [22]

Inverse kinematics for manipulators: determines the joint angles from the required end-effector location and orientation; this is the inverse kinematics issue [23]. Complex mathematical formulations based on trigonometry, matrix operations, and numerical approaches may be used to find solutions for the joint angles ($\theta_1, \theta_2, \ldots, \theta_6$) in a generic 6-degree of freedom (6-DOF) robotic manipulator.

3.4. Advanced modeling and simulation techniques
Engineers and researchers may now analyze complicated mechanical systems with unprecedented fidelity and detail because to the advent of cutting-edge modeling and simulation tools. Here, the review article delves into some of the most cutting-edge methods for simulating and modeling complicated mechanical systems and their performance characteristics.

3.5. Finite element analysis
The behavior of complicated structures and its constituent parts may be simulated and analyzed with the help of finite element analysis (FEA), a sophisticated numerical tool. One computer approach to approximating engineering issue solutions is using FEA. As the name implies, numerical approaches are used to analyse the behaviour of individual "finite elements" within a larger complex geometry or system. Engineers may learn a lot about the system's overall performance under various situations by integrating the behaviours of all the pieces. Engineers may use FEA to optimize designs by analyzing the mechanical system's stress distribution, deformation, and failure mechanisms [24].

3.6. Computational dynamics
To solve the equations of motion for elaborate mechanical systems, computer techniques like numerical integration and differential equations are used in the field of computational dynamics. System responses to varying inputs, loads, and boundary conditions may be predicted. The dynamic behavior of machines, manipulators, and transport machines is greatly aided by computational dynamics methods [25].

Considering the four bar concept the analysis behind it accumulates sparsity pattern shown in Figure 4. It can be seen that the larger system of linear equations to be solved is based on a lower triangular matrix; this matrix corresponds to the open chain system. The closure condition of the loop introduces a small system of three equations. It can be useful to remember that the Jacobian of an open chain system can always be arranged in a triangular form, that of course do not need to be factorized. It is also worth to remember that forward and backward substitutions with a sparse triangular matrix are computationally equivalent to forward and backward recursive processes.
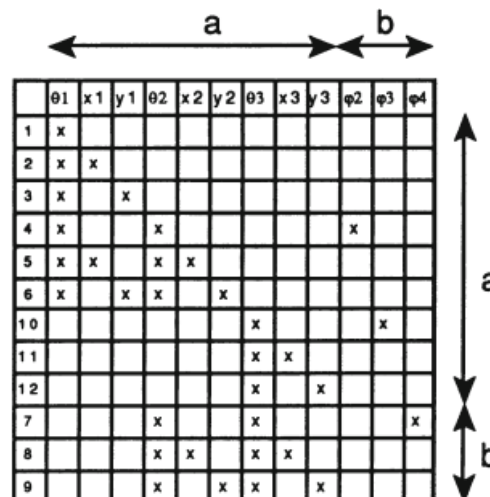

Figure 4. Sparsity pattern for the mechanism for Figure 1

3.7. Multibody dynamics simulation
To determine the motion and stability of mechanical systems, a multibody analysis based on the rigid body dynamics technique is applied [26]. A component's stresses and deformation may be computed using the forces it was subjected to in retrospect. For elastic deformations of moderate size, this method is applicable [27]. However, a flexible multibody analysis is the only approach to achieve precise findings and, therefore, optimize the design in the presence of substantial deformations and material nonlinearities [28].

Certain parts of a system are treated as flexible in a flexible multibody analysis while other parts are deemed rigid. The new multibody dynamics module in COMSOL version 4.3b makes it simple to create connections using joints and run a flexible multibody analysis. This package contains an assortment of joints, including prismatic, hinge, cylindrical, screw, planar, ball, slot, and reduced slot joints, to name a few. An array of multibody dynamics investigations, including transient, frequency-domain, eigenfrequency, and stationary, are available [29].

3.8. Optimization methods

Genetic algorithms, simulated annealing, and gradient-based approaches are only few of the optimization strategies used to identify the optimal design parameters for optimizing goals like maximum performance and minimal weight. Optimization techniques play a key role in increasing productivity, decreasing expenses, and satisfying design restrictions in complicated mechanical systems (Figure 5).
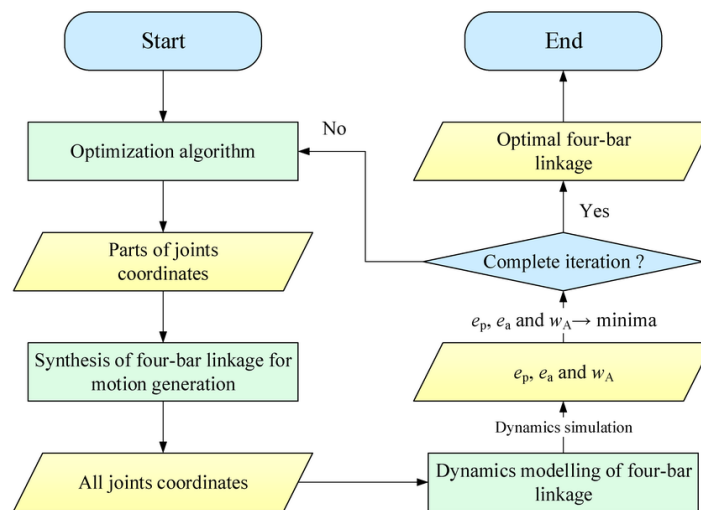

Figure 5. Flowchart of design framework [30]

The optimisation procedure begins by generating joint coordinates for the four-bar connection based on the total number of task locations. Then, after the joint coordinates have been collected, the four-bar linkage synthesis for motion production is carried out. These joint coordinates may be used to simulate the dynamics of a four-bar connection. The parameters ε_p, ε_a, and w_A may be calculated by simulating the dynamics of the system. Here, we want to optimise for the global minimum of ε_p, ε_a, and w_A. After the optimisation iteration calculation is complete, the Pareto-optimal set of joints coordinates for optimisation variables may be retrieved [31], [32]. Additionally, the best possible joint coordinates for a four-bar connection are determined. At last, we can zero in on the best configuration for a four-bar connection.

4.   RESULTS AND DISCUSSION

It is not possible to begin a dynamic simulation without a bunch of beginning circumstances (positions and velocities). Predicting the dynamic behavior of a mechanical system relies heavily on choosing suitable beginning conditions [33]. In this work, we use the outcomes of a kinematic simulation of a mechanical system in which all the joints are considered to be ideal-that is, without clearance-to establish the beginning circumstances. The next time step's beginning conditions are produced in the customary fashion from the preceding time step's final conditions. The Baumgarte stabilization approach [34] is used to solve in an effort to bring the constraints violation under control or to stabilise it. To that end, an adaptable step size and order predictor-corrector [35], [36] is used to carry out the integration (Figure 6).

Finally, it's important to assess how much CPU time was used throughout each simulation. Figure 7 displays the amount of computational time used as a function of (a) diametric clearance size, (b) input crank speed, and (c) the number of clearance joints modelled. Figure 7(a) shows us immediately that the system's computation time decreases as the number of clearance joints increases. Secondly, Figure 7(b) the clearance size has a favourable influence on calculation time, decreasing it as the number of simulated clearance joints increases. There is little effect on calculation time from small changes in input crank speed. And lastly

Figure 7(c) considers the number of clearance joints modelled within the area recommending the computational timeframe. We used the multibody dynamics (MUBODYNA) program for our computational calculations. This software is designed to perform dynamic analysis of typical planar multibody systems [37].
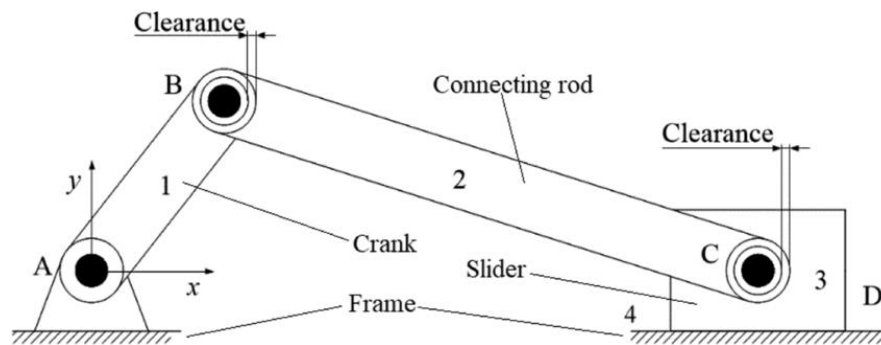
Figure 6. Clearance between the two revolving joints of the slider-crank mechanism [36]
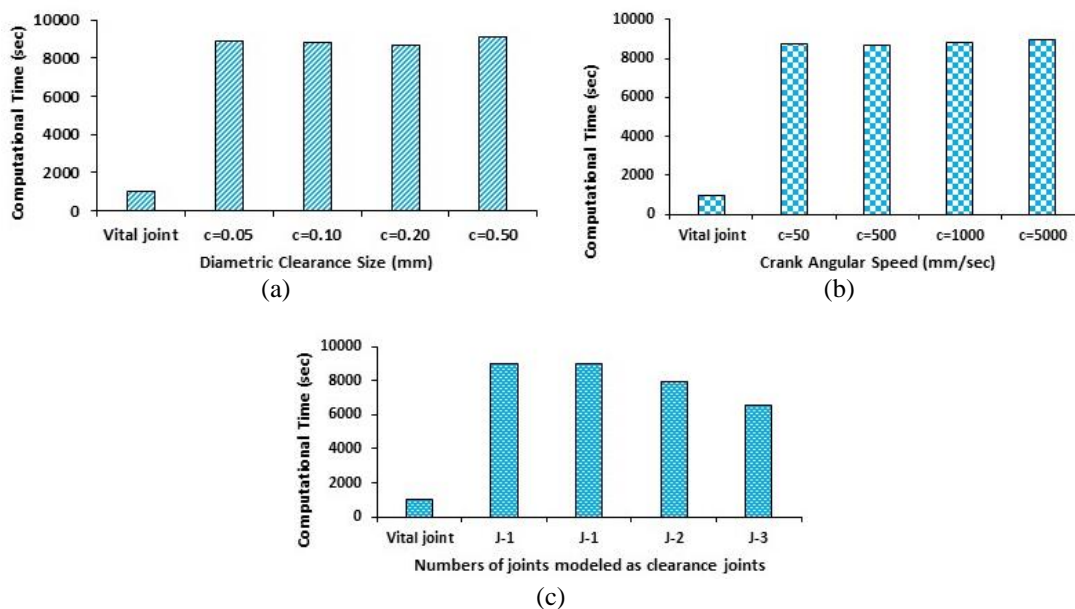
(a)

(b)

(c)

Figure 7. Variation in the amount of time spent computing, (a) diametric clearance size, (b) input crank angular speed, and (c) number of joints with clearance [37]

4.1. Discussion

The findings of this study on the dynamics and kinematics of complex mechanical systems, including machines, manipulators, and others, provide important new perspectives on how these sophisticated systems behave and function. Our research adds to the corpus of knowledge by illuminating important facets of system behavior and providing fresh viewpoints on the conception and improvement. The findings support various accepted theories about the dynamics and kinematics of complex mechanical systems when compared to earlier research.

The practical implications of the results for improving the performance and dependability of complex mechanical systems are what make them significant. The design can be improved, also the control methods, and safety features of machines, manipulators, and other systems by having a better grasp of dynamics and kinematics. The study further emphasizes the significance of developing modelling methodologies to precisely capture nonlinear factors including friction, contact interactions, and system coupling. This realization has larger ramifications for areas outside of the direct area of study, such as robotics, aircraft, and manufacturing, where it is crucial to accurately model complicated mechanical behavior.

Contrary to expectations: the surprising vibration patterns found in our transport machine investigation demonstrate how mechanical interactions are complicated and sometimes nonlinear. This departure from our original predictions emphasises the necessity for thorough modelling that takes a broader variety of variables into account. These could consist of material qualities, system resonances, and real-world variances that simplified models might not adequately account for.

In conclusion, the study of the dynamics and kinematics of intricate mechanical systems broadens our comprehension of their functioning. Our results support well-established theories while also highlighting the need for sophisticated modelling methods to accurately represent complex behaviors. These discoveries directly affect complicated mechanical system design optimization, control strategy improvement, and overall dependability across numerous sectors.

5. CONCLUSION

The study of complicated mechanical systems, such as machines, transport machines, manipulators, flat and spatial mechanisms, and gears, has benefited greatly from the information presented in this review work. The importance of dynamics and kinematics analysis in contemporary engineering has been elucidated via the study of basic principles, mathematical expressions, and cutting-edge modelling and simulation tools. The importance of mastering the concepts of angular and linear kinematics, inertial characteristics, and the interaction of forces and torques in determining the behavior of complicated mechanical systems was emphasized throughout the review. We looked at the kinematic analysis of flat and spatial mechanisms, machine and transport machine dynamics, and the complexity of manipulators and gears to get a full picture of their capabilities and design constraints. The continuous contact force concept supplies the contact-impact forces produced inside a joint during typical mechanism operation. The multibody systems approach incorporates an appropriate model for revolute clearance joints. Finally, it should be stressed that the results published in this work show an upper limit of the joint reaction forces and crank moments due to the existence of clearance joints, because the elasticity of bodies and the lubricating action in the joints were not accounted for in the research.

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BIOGRAPHIES OF AUTHORS

Mykhailo Kaidash [iD] [SC] ⟳ Ph.D. in Engineering, Professor of the Department of Welding and Construction Technologies, Chernihiv Polytechnic National University, Chernihiv, Ukraine. He can be contacted at email: 0203kmdcn@gmail.com.

Serhii Selevych [iD] [SC] ⟳ Ph.D. in Engineering, Associate Professor of the Department of Car and Tractor Industry, Institute of Education and Science in Mechanical Engineering and Transport, National Technical University (Kharkiv Polytechnic Institute), Kharkiv, Ukraine. He can be contacted at email: sergej.selevich@gmail.com.